LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – NOVEMBER 2007

MT 1806 - ORDINARY DIFFERENTIAL EQUATIONS

AB 21

Date : 31/10/2007 Dept. No. Max. : 100 Marks
Time : 1:00 - 4:00

ANSWER ALL QUESTIONS

I. a) Consider the Differential Equation $x'' + \lambda^2 x = 0$, prove that $A \cos \lambda x + B \sin \lambda x$ is also a solution of the Differential equation.

OR

If the Wronskian of 2 functions $x_1(t)$ and $x_2(t)$ on I is non-zero for at least one point of the interval I, show that $x_1(t)$ and $x_2(t)$ are linearly independent on I. (5 Marks)

b) State and prove the method of variation of parameters.

OR

By the method of variation of parameters solve $x''' + x'' + x' + x = 1$

(15 Marks)

II. a) Prove that $J_n'(x) = J_{n-1}(x) - (n/x) J_n(x)$

OR

Show that the generating function for the Legendre polynomial is

$(1/\sqrt{1 - 2tx + t^2}) = \sum_{n=0}^{\infty} t^n P_n(x)$ if $|t| < 1$ & $|x| \le 1$ (5 Marks)

b) Solve the Bessel's equation $x^2 y'' + xy' + (x^2 - n^2) y = 0$

OR

Solve $9x(1 - x) y'' - 12 y' + 4y = 0$ (15 Marks)

III. a) Using Rodrigues' Formula, find $P_0(x)$, $P_1(x)$, $P_2(x)$ & $P_3(x)$.

OR

Show that $F(1; p; p; x) = 1/(1 - x)$ (5 Marks)

b) Show that Gauss' equation has $2F_1(\alpha; \beta; \gamma; x)$ as a solution.

OR

State and prove the Integral representation of $2F_1(\alpha; \beta; \gamma; x)$. (15 Marks)

IV. a) Considering the Differential Equation of the Sturm-Liouville Problem, prove that all the eigen values are real.

OR

Solve the initial value problem $x' = 2t - x$, $x(0) = 1$ (5 marks)

b) State and prove Picard's Boundary Value Problem.

OR

State Green's Function. Prove that x(t) is a solution of $L(x) + f(t) = 0$ if and only if $x(t) = \int_a^b G(t,s)\, f(s)\, ds$. (15 Marks)

V. a) Give examples of Lyapunov's Stability definitions.

OR

Obtain the condition for the null solution of the system $x' = A(t)\,x$ is asymptotically stable. (5 Marks)

b) Study the stability of Autonomous Systems $x' = g(x)$.

OR

Study the stability of $x' = A\,x$ by Lyapunov's Direct Method.

(15 Marks)

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